# ASTROMETRIC DETECTION OF EXTRA-SOLAR PLANETS: RESULTS OF' A FEASIBILITY STUDY WITH THE PALOMAR 5-M


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## ABSTRACT

The detection of extra-solar planets around stars like the Sun remains an unfulfilled goal of astronomy. We present results from Palomar 5-m observations of the open cluster NGC **2420** in which we measure some of the sources of noise that will be present in an astrometric search for extra-solar planets. We find that the atmospheric noise is 150 $\mu as \sqrt{hr}$ across a 90 arcsec field of view, and that differential chromatic refraction (DCR) can be calibrated to 128 $\mu as$ for observations within 1 hr of the meridian and 450 of zenith.

These results indicate that a large telescope achieves the sensitivity required to perform a statistically significant search for extra-solar planets. We describe an astrometric technique to detect planets, the astrometric signal expected from the target stars in the solar neighborhood, and the sources of measurement noise: photometric noise, atmospheric motion between stars, sky background, instrumental noise, and DCR. For the latter we discuss a method to improve our current results and reduce the noise to 66 $\mu as$ for observations within 1 hr of the meridian and 450 of zenith. Two sample programs are described which can perform statistically significant searches for gas-giant planets around nearby stars using a CCD camera on a 10-m telescope in 40 nights $yr^{-1}$ of observations. One program uses 100 "solar-class" stars for targets with an average stellar mass of 0.75 MO; the other maximizes the number of stars, 574, by searching mainly low-mass M stars. The target stars are taken from the latest Gliese & Jahreiss catalog of nearby stars, and are chosen for: the largest potential astrometric signals, declination limits for both telescope accessibility y and reduced DCR, and galactic latitude limits for a sufficient number of reference stars, We perform Monte Carlo simulations of the statistical significance of the expected results by using measured and estimated noise quantities. We show the semi-major axes parameter spaces that are searched for each star and how an increase in the length of the observing program expands these spaces. Finally we discuss how the search over semi-major axes parameter space connects with the theory of gas-giant planet formation.

*Subject headings:* stars: planetary systems, astrometry: instruments

# 1. INTRODUCTION

## 1.1 Background

The question "Do other stars have planets?" is of interest to scientists and non-scientists alike. The answer in scientific terms is not easy, however, because even if extra-solar planets are ubiquitous no one has yet successfully detected one. Continuing observations of nearby stars using various techniques including astrometry (e.g. Gatewood 1987) and radial velocity measurements (e.g. Latham **et al. 1989,** Cochran **et al. 1991,** Marcy et al. 1993, McMillan et al. 1994) have to date yielded only upper limits to the masses of possible planets. Some false alarms of planet detections have turned out to be discoveries of low-mass binary companion stars. other inferred planet detections (Marsh and Mahoney 1992, 1993) are subject to alternative interpretat ions (Boss& Yorke 1993). The detection of planets around a pulsar (Wolszczan & Frail 1992) is on firmer ground (but see Gil et al. 1993, Peale 1993), but may be unrelated to the solar-like planetary systems that we consider herein. In summary, as of now, there are no known planets orbiting main sequence stars other than the Sun.

Both theoretical models and observational evidence lead us to conclude that the solar system is not the result of an improbable path of stellar evolution and therefore, that planetary systems are common. Black & Matthews (1985) and Levy et al. (1990) compile much of the theoretical work on solar system formation and the observational evidence. The observations describe an evolutionary chain from protostars, through pre-main sequence stars, to stars like the Sun. Circumstellar disks (e.g. Smith & Terrile 1984) area common phenomenon in young stars which maybe the precursors of planetary systems.

The planets are believed to have formed in the protosolar nebula (e.g. Safronov 1969). The solar system is organized into two types of planets: gas giants (Jupiter, Saturn, Uranus, Neptune) and terrestrial planets (Earth, Mars, Venus, Mercury). Gas giants form beyond the water-condensation radius, a distance from the center of the nebula beyond which water and other abundant volatiles remain solid (e.g. Safronov & Ruskol 1994), while terrestrial planets form within this radius. Pluto is an anomaly. Gas giants are also 10-300 times more massive than terrestrial planets because more planet-forming material is available at their locations. Based on the solar model, the astrometric search for gas giants is easier than for terrestrial planets because they are more massive and further from the star, both contributors to larger astrometric signals.

## 1.2 Extra-Solar Planet Detection

Extra-solar planet detection is a unique task because the best targets are well known and unlikely to change. Other than low-mass M stars, most of the stars within 25 pc of the Sun are known. It remains only to implement a detection technique with a systematic program to find planets assuming they exist.

‘I’he astrometric signals from the putative planetary systems are well-defined. They depend only on the planet-star mass ratio, the planet-star separation, and the distance to the star. The sources of noise are many. In the visible, using a ground-based single-aperture telescope and an area detector they are: photon noise, atmospheric noise, background noise, uncalibrated differential chromatic refraction (DCR), detector noise, uncalibrated geometric errors on the detector, and uncalibrated optical aberrations. We have undertaken a program to measure the sources of noise which can not be modeled reliably. For example, Shaklan et al. (1995) demonstrate a method to calibrate geometric errors on CCD pixels to a level of 0.01 pixels. Herein we present results from Palomar 5-m observations that are relevant for atmospheric noise and DCR.

Atmospheric noise may be the limiting factor for high-precision ground-based astrometry. Lindegren (1 980) shows that the astrometric motion between two stars separated by angle $\theta$ due to the atmosphere is proportional to $\theta^{-1}$ in the narrow-angle regime: for large apertures the narrow-angle regime extends to several arcminutes. The motion also varies with aperture diameter as D $^{-2/3}$ (Shao & Colavita 1992, hereafter SC). Observations confirm this behavior (Gatewood 1991, Colavita 1994, Dekany et al. 1994). SC give this equation for narrow-angle differential astrometry oft wo stars (1 arcsec seeing):

$$\sigma_a \approx 540\ D^{-2/3}\ \theta\ t^{-1/2} \tag{1}$$

where $\sigma_a$ is the atmospheric noise or standard deviation in arcsec of a d inferential measurement, $D$ is the telescope diameter in meters, $\theta$ is the field of view in radians, and $t$ is the integration time in seconds. For example, with D = 5 m, $\theta = 1'$, $\sigma_a = 54$ *mas* $t^{-1/2}$. To detect a Jupiter-mass planet in a 5 au. orbit around a solar-mass star at 10 pc would require, according to this prevailing theory, a precision better than 1 *mas* and thus a time of at least 2900 s. Our measurements described below show that the atmospheric noise term is over-estimated in **(1)** and the time to achieve a given precision is correspondingly less.

This paper is organized in the following way. First, we present Palomar 5-m observations of the open cluster NGC 2420 and discuss our results. We then describe the astrometric technique for planet detection and present new estimates of the measurement noise based upon our 5-m results. Last, we describe a new experiment to survey two samples of nearby stars for planets using a large-diameter ground-based telescope. The search for planetary systems is contributing to the development of many new instruments and techniques designed to solve the difficult problem of planet detection (Burke et al. 1993). In this paper we discuss a program that has the potential for solving this problem and we present observational evidence that this program will work.

## 2. OBSERVATIONS

We observed the open cluster NGC 2420 on 5 Feb 1995 at the f/16 Cassegrain focus of the Palomar 5-m (200 in.) telescope using the CCD-13 camera. CCD-13 is a SITE 2048 x 2048 backside-illuminated device with 24 μm square pixels. The camera is attached to the f/9 base and guider assembly (McCarthy 1994). The f/9 optics were not used.

NGC 2420 is a high galactic latitude cluster that was chosen because of its position, passing within 90 of zenith and transiting at Oh local sidereal time. It contains a sufficient density of bright stars to permit an astrometric study that is atmospherically, rather than photometrically, limited. Detailed photometric analysis of this cluster has been carried out by Anthony-Twarog et al. (1 990 hereafter AT). Table 1 identifies the stars used in our analysis, their magnitude, and color using Table 2 of AT. The identification numbers are due to West (1967). Figure 1 is a plot of the positions of the stars with their West identifications.

The pixel size on the CCD is 24 μm resulting in a pixel scaleof61 milli-arcsec (*mas*) pixel-l, and a field-of-view (FOV) 124 *as* wide. The CCD was oriented with RA increasing along columns and Declination decreasing along rows. The CCD showed no evidence of bad columns, but the lower left portion of the field was obscured by the guide camera periscope, which had frozen in place early in the evening. Stars that may have been partially vignetted by the periscope were not used in this study. The CCD read noise was 6,7 e-, and its gain was set to 1.8 $e^-$/DN.

The cluster was observed from 19 minutes East to 3 hours West of meridian. A set of 55 1-minute exposures was made at -3.3 minute intervals. 'l'he position of the cluster on the CCD was not moved during the first 24 exposures. During the 25th exposure, a pointing error caused by slippage of the guider periscope resulted in a 5 *as* shift of frames 26-55 compared to 1-24. As explained below, the two sets can not be accurately compared at the 0.1 *mas* level. Thus, we analyze the data in two sets, A, and B, representing frames 1-24 and 26-55.

Atmospheric conditions were excellent and at times yielded sub-arcsecond seeing. During the observations the outside temperature decreased by 0.2° C while the dome air, which was nominally 2° C cooler, warmed by 0.10 **C.** The observations were made on the second night of a stable 3 day weather pattern. The sky was clear except for cirrus near the west horizon.

'I'he astrometric observations were made using a bandpass color filter composed of a long-pass and short-pass filter placed in series in the filter wheel of the f/9 base. The filters were manufactured by CVI Laser Corp. on plate glass. The long-pass filter has a cut-on wavelength of 550 nm, with attenuation >99% below 525 nm and full transmission at 575 nrn. The short-pass filter has a cut-off at 750 nm, with full transmission to 740 nm and attenuation > 99°/0 beyond 775 nm. Because they are built on plate glass, local gradients in the wavefronts passing through the filters result in positional errors > 1 *mas*. This is why our experiments required that the stars remain stationary to -1 arcsec during the observations. The 5 arcsecond guiding error in frame 25 caused one star to be displaced by 6 *mas* relative to the other stars. Future astrometric programs will require accurately calibrated, flatter filters.

We also observed the cluster through three bandpass filters. These were centered at **550, 650, and 750** nm, each with a fill-width-at-half-maximum (FWHM) of 40 nm. The purpose of these observations was to calibrate the differential chromatic refraction in the frame. This will be further explained below.

## 3. DATA REDUCTION

The astrometric reduction of the frames followed four steps: centroiding, removal of differential chromatic refraction (DCR), fitting with a plate-scale model, and computation of the AlIan variance for each star, DCR is removed from the data before the data is fitted with a linear model. However, as described below, residual DCR (due to poor calibration) is observed.

### 3.1 Centroiding

Centroids were computed by centering a 4.9x 4.9 arcsecond (80x 80 pixel) box on each of the 15 stars. Two types of centroids were computed within the box: the first moment of the marginal distribution, and a Gaussian fit to the image. The moment was chosen because of its simplicity and insensitivity to aberrations. The Gaussian was chosen because its performance has been shown to be competitive with several more complicated algorithms as demonstrated by Stone (1989).

The moment was computed (for the row direction) using

$$x_c = \frac{\Sigma I_{ij}.i - B}{\Sigma I_{ij} - B} \qquad (2)$$

The background $B$ was determined for each frame by averaging wo *Gas x 6as* regions without detectable stars on the CCD. Unlike Stone(1 989), images were not rimmed to increase centroiding precision for fear of loss of accuracy.

An elliptical Gaussian function was also fitted to the images. Parameters of the fit were the amplitude, x and y center, position angle of the major axis, major and minor axial widths, and background level. The procedure used was to fit all 7 parameters to star 1116, then fix the shape parameters, fitting only amplitude, x and y center, and background on the remaining stars.

In practice we found that the first moment performed as well on average as the Gaussians. Each resulted in a frame-to-frame standard deviation of -1 *mas*. This is consistent with the observed centroid noise being largely due to atmospheric statistics rather than photon and background statistics. The first moment variances still show dependence on source brightness, **while the Gaussian** performance appears to be limited by systematic effects, This is due in large part to the mismatch between the symmetrical Gaussians and the non-symmetrical images, The mismatch led to a 10°/0 fitting error at the image peak, and a poor estimate of the image wings. For observations of background-limited sources, an improvement on the Gaussian fits would be required, But for the purposes of this analysis, in which the centroid noise is largely dominated by the atmosphere, the simple first moment approach is adequate. In what follows, only the centroids obtained by the first moment calculation are discussed.

### 3.2 Correction of Differential Chromatic Refraction (DCR)

DCR results in relative astrometric errors (e.g. Monet et al. 1992). As stars move away from the meridian, the bluer stars in the frame appear to shift toward the zenith, while the red stars appear to shift toward the horizon. This occurs because bluer stars refract more than red stars at non-zero zenith angles. We calibrate DCR for our reference fields in the following way. We observe broadband in the visible avoiding water absorption bands in the near infrared, We then estimate the blackbody temperatures of the target and reference stars in the field by observing them with the three bandpass filters. Note that these filters fall within the astrometric filter band.

The effect is quadratic in wave number. A fit to the data in Allen (1973) for $\lambda$ = 500-800 nm yields the refractive constant $R(\lambda)$ for an air temperature of O°C and pressure of 760 mm Hg:

$$R(\lambda) = 59.24 + \frac{346466}{\lambda^2} \text{ arcsec} \qquad (3)$$

where the wavelength is expressed in nm. The refractive constant changes by hundreds of milli-arcseconds across the visible and near IR spectrum. It is thus critical to accurately characterize the refractive constant of the stars to remove color-dependent centroid shifts. The temperature dependence of $R(\lambda)$ is - 4% for a 10°C change.

The effective refractive constant for a star, $R_{eff}$, depends on the spectrum of the star, $I(\lambda)$, the bandpass set by the filter, $T(\lambda)$, and the CCD quantum efficiency, $Q(\lambda)$. The refractive constant is given by the normalized product of these quantities:

$$R_{eff} = \frac{\int R(\lambda)T(\lambda)Q(\lambda)I(\lambda)d\lambda}{\int T(\lambda)Q(\lambda)I(\lambda)d\lambda} \quad (4)$$

$Q(\lambda)$ is obtained from the calibrated CCD-13 response. $T(\lambda)$ is between 550-750 nm and is quantitatively described by the filter specifications. The total refraction observed is given to first order by $R_{eff}\tan(z)$ where z is the apparent zenith angle.

We do not independently know $I(\lambda)$ for many of the program stars. Instead, we use an approximation for $I(\lambda)$ determined for each star from its intensities measured through the three narrow bandpass filters. These data are divided by the filter transmission functions and CCD quantum efficiency, and fitted to blackbody curves. The only parameter of this fit is the blackbody temperature. Once fitted, the blackbodies are used as $I(\lambda)$ in (3) to determine $R_{eff}$.

Weusedstandard spectra ofseveral dozen stars ofallspectral types (Jacobyet al. 1984) to estimate the effectiveness of this technique. The above procedure was applied to the stellar spectra and the refractive constant derived from the blackbody fit was compared to that obtained for the stellar spectrum. For dwarf stars ranging in spectral type from 134 to MS, the estimated refractive constant had a standard deviation of 0.429 *mas* when comparing the true and modeled $I(\lambda)$.

For the NGC 2420 observation the narrow bandpass filter transmissions were calibrated using standard stars with an accuracy of 2°/0. This is not a critical measurement because all stars are observed simultaneously through the same filter. Filter calibration errors then have a negligible effect on the relative blackbody temperature estimates, and thus the relative DCR of all stars in the field of view.

Once the'refractive constants are determined, the estimated DCR centroid shifts are removed from the measured centroid positions. As the temperature was constant to 0.2 "C during the observations, no thermal corrections were applied to the estimated DCR terms. We emphasize that since only relative astrometry within a frame is performed, the temperature dependence of the calibration depends only on the second term of (2). For example, given a star with a refractive constant of 10 *mas* relative to the other stars in the frame, a 10" C change during the observations leads to 0.4 *mas* error in its position. A more detailed description of the temperature and pressure dependence of the DCR correction is given by Monet et al. (1992).

### 3.3 **Plate Scale Model**

The corrected centroids were next fitted frame-by-frame to a linear plate-scale model with 6 linear constants defined by

$$x(i,r) = a_f\, x(i,f) + b_f\, y(i,f) + c_f \tag{5}$$

$$y(i,r) = d_f\, x(i,f) + e_f\, y(i,f) + g_f \tag{6}$$

A reference framer was chosen, so that each star *I* in frame *f* was fitted to star *I* in frame *r* using a least squares solution where all stars received equal weight.

### *3.4* **AlIan Variance**

The astrometric program discussed in the later sections of this paper relies on a $\sqrt{t}$ improvement in astrometric precision to achieve the sensitivity required to detect extra-solar planets. Atmospheric models predict that differential astrometric measurements exhibit a white power spectrum and therefore improve as $\sqrt{t}$ for inter-frame $t \geq 1$ *s (SC).* We compute the Al Ian (1966) variance for the atmospheric noise to characterize the improvement in astrometric precision versus integration time. The calculation also indicates the level of systematic errors in the data, The AlIan variance for the combined atmospheric and photometric noise is given by (SC):

$$\sigma_x^2 = \frac{1}{2(M+1-2l)} \sum_{n=0}^{M-2l} \left( \frac{1}{l} \sum_{m=1}^{l-1} x_{n+m} - x_{l+n+m} \right)^2 \tag{7}$$

where $M$ is the total number of frames, $l$ is the lag, and x is the measured stellar position. The atmospheric noise is derived from $\sigma_a^2 = \sqrt{(\sigma_x^2 - \sigma_p^2)}$ where $\sigma_p$ is the positional error due to photometric noise. For $\sqrt{t}$ behavior, the Allan variance has a slope of-1 on a log-log plot of centroid motion versus integration time. The slope flattens and can eventually reverse itself when systematic errors become significant with respect to the noise in time-integrated data.

## 4. RESULT'S

After computing the first moment centroids for each star, the positions were corrected for DCR. The refractive constant $R_{eff}$ was determined using the 3-filter method described above, then multiplied by tan(z) and subtracted from the measured centroids. Column 6 of Table 1 gives $R_{eff}$ for each of *the* 15 stars. For the frames in set B, tan(z) ranged from 0.38 to 0.90, yielding refractive corrections that changed by 8.4 *mas* for star 1116, and -6.08 *mas* for star 2122. The accuracy of the corrections is estimated to be 0.43 *mas* at tan(z) = 1 ($3.2).

The corrected positions were then fitted using the linear plate-scale model. The model shows a significant discontinuity between data sets A and B of -4 *mas*. The discontinuity is not related to either CCD or telescope distortion. While CCD step-and-repeat errors of 0.5 microns have been

The AlIan variance was computed for each star. The variances are biased by photometric noise due to photon statistics, read noise, and sky background. We estimated the photometric noise bias by employing a Monte Carlo simulation; we first took an image of the star from one of the flames, then added background noise, read noise, and photon noise according to the flux of each star. Background and read noise were estimated by comparing pixels in several frames from a starless region of the frame. We then computed the first moment, and repeated the Monte-Carlo procedure 100 times for each star. The root-mean-square (rms) phot ometric bias and de-biased frame-to-frame centroid rms are given in Table 2. The rss value of the de-biased Dec. data (Table 2 column 6) is 0.37 *rims*. This result, when extrapolated to a 1 hr integration and a 90" field-of-view, yields 150 $\mu as$ $\sqrt{hr}$ noise.

The square root of the de-biased Allan variance foı the 15 stars is plotted in Figure 2. These curves are for data set B, containing 31 one-minute exposures. The frames cover hour angles $1^{h}14^{m}$ to $3^{h}3^{m}$. Each curve represents the improvement in astrometric precision of one star. The two dashed lines indicate $\sqrt{t}$ behavior. The values at the left edge of the plots can be found in columns 3 and 4 of Table 2. The root-sum-square (rss) noise for all one-minute exposures is 0.82 *mas* in RA and 1.39 *mas* in Dec. When grouped into 10 minute integrations, the rss value is 0.23 *rnas* in RA, 0.37 *mas* Dec. These numbers show that the mean astrometric precision is improving by $\sqrt{t}$, and that precision of 0,23 *mas* can be achieved without systematic limitation.

The curves do not all decrease monotonically with integration time. This has been shown through Monte-Carlo simulations of the observations to be consistent with the finite number of frames and stars, and is not indicative of systematic limitations. The spread of the curves is, however, inconsistent by a factor of 2 with stars all having the same underlying centroid noise. The spread shows that either photometric biases have not been properly removed, and/or the noise depends on the position on the CCD (or field angle). In either case, the excess noise is additive and leads us to a slightly conservative analysis of atmospheric turbulence limitations. As seen in the next section, even this conservative view is substantially better than the expected results based on standard atmospheric models.

## 5. EXPERIMENTAL CONCLUSIONS

Our 5-m results indicate that the atmospheric noise is smaller than expected for a large aperture and small field. The rss positional errors of the stars after 10 minutes of integration are significantly better than one would have expected from standard atmospheric models assuming 1 *as* seeing. According to (1), the standard deviation should be 1.3 *mas* for a 36 *as* field of view (as we have along the RA axis), a 5-m aperture, and 10-minute integration, whereas we obtained 0.23 *mas*. Along the Dec axis, which extends 91 arcsec, the expected noise is 3.3 *mas*, compared to our result of 0.37 *mas*. We thus see an improvement of a factor of -6-9 over the predictions.

Part of the difference between the expected and measured results is due to the plate-scale model that is fit to each frame. The major atmospheric effect on differential astrometry is plate-scale variation, which is removed from the data on a frame-by-frame basis, The ability to remove plate-scale variation and possibly higher order terms is a clear advantage of a single aperture system with several reference stars compared to an interferometer which measures one reference star at a time.

To demonstrate the effect of turbulence without removing frame-to-frame plate-scale variation, we performed a second data reduction in which the plate scale constants *a, b, d,* and e were first fitted by a quadratic function of the form $a = a_o$ -t $a_q t^2$ where *t* is the time of observation. This provided correction for gradual telescope defocus and sag, as well as differential atmospheric refraction (non-chromatic) bet ween azimuth and elevation. The resulting standard deviations were 0.38 *mas* in RA and 0.47 *mas* in Dec. The values are still -3-7 times smaller than expected from atmospheric models.

The unexpectedly good astrometric performance of the S-m is explained by attributing much of the atmospheric turbulence to dome seeing. Dome seeing is common to all stars in the field: the "isokinetic patch" in which all stars have similar motion, extends over many arcminutes. The standard Hufnagel model (Hufnagel 1974) used by SC appears to put too much weight on high-altitude seeing for our observations. The Hufnagel model describes the atmospheric structure constant, $C_n^2$, with two terms; one extends to an altitude of about 5 km, and the other is a broad peak at 10 km. The seeing disk is proportional to $\int C_n^2 dh$ while the variance of differential astrometric measurements depends on $\int C_n^2 h^2 dh$. Clearly, astrometric measurements are most affected by high altitude turbulence. If the model is modified by removing the high altitude term and increasing the low-altitude term, the seeing disk diameter remains constant (at about 1*as*) while the differential astrometric standard deviation is reduced to 0.3 of the standard-model result, Thus, by reforming the model to emphasize dome and low-altitude seeing while de-emphasizing high-altitude seeing, the expected atmospheric noise is reduced from 1.3 *mas* to O.39 *mas*. This is now in agreement with our observations when frame-to-frame plate-scale terms are not removed.

In summary, our observations show that the standard atmospheric model places too much emphasis on high-altitude seeing for the Palomar 5-m telescope. The results show that, after frame-by-frame removal of the linear plate-scale terms, the differential astrometric noise is ~6 - 9 times better than expected. This greatly increases the potential for this telescope to carry out an astrometric search for extra-solar planets. We next insert our value for astrometric noise into an analysis of sample planet detection programs.

## 6. AN EXTRA-SOLAR PLANET DETECTION PROGRAM

### 6.1. Planet Detection Technique

Planets are detected by astrometrically measuring the motion of the stars around their planetary system center-of-mass. Consider the simplest case of a single planet orbiting a single star. Then the astrometric signal $\Theta$ of the star is:

$$\Theta\,(\mu as) = 96 \left( \frac{a}{5a.u.} \right) \left( \frac{d}{10pc} \right) \left( \frac{M_p}{M_J} \right) \left( \frac{M_\odot}{M_S} \right) \tag{8}$$

where $a$ is the semi-major axis of the planetary orbit, $d$ is the distance to the star, $(M_P/M_J)$ is the planet-to-Jupiter mass ratio, and $(M_S/M_\odot)$ is the stellar mass in solar units (Figure 3). The period $T$ of the planet is:

$$T(yr) = 11 \left( \frac{u}{5a.u.} \right)^{1.5} \left| \frac{M_S}{M_\odot} \right)^{0.5} \tag{9}$$

A current astrometric program (Gatewood et al. 1990) achieves accuracies on the order of 1 *mas* and is thus capable of detecting Jupiter-like planets around some stars. Note that the orbital period is an important observational parameter since it defines the observing program length and, for fixed $M_S$, $\Theta$ is proportional to $7^{0.67}$.

We measure the position of a target star relative to a surrounding field of reference stars. These stars form a fixed frame of reference from which changes in position of the target star are determined. This technique was described by Eichhorn & Williams **(1963)** and is employed by Gatewood (1987), The reference stars allow us to make an affine transformation between observations (e.g. Eichorn & Williams, Shaklan et al. 1994a). At least three reference stars are required to generate a linear model of the field; six reference stars for a quadratic model. Since we also solve for the parallaxes and proper motions of the reference stars, an additional reference star is required **for either** model. With the small fields discussed below ($7.1) a linear model is usually sufficient. In any case with more reference stars, the field model is more robust. The field model also accounts for optical aberrations ($6.4).

The astrometric data $\Sigma$ from the target star corrected to the solar barycenter and, which we simplify from two dimensions to one, can be written:

$$\Sigma(t) = \left(\frac{\Theta}{2}\right)\sin\left(2\pi\frac{t}{T} + \phi\right) + \mu t + P\sin(2\pi t) + c + \sigma_o \qquad \textbf{(10)}$$

where $t$ is the time of observation, $\Theta$ is the amplitude or signal, $\phi$ is the orbital phase at time zero, $\mu$ is the proper motion, $P$ is the parallax, and c is the mean position, $\sigma_o$ is the standard deviation of the noise per observation. It is estimated from the data by determining the exposure-to-exposure target star motion relative to the reference stars. One can detect a planetary signal in a time sequence of such data in a number of ways including Fourier or periodigram (Black & Scargle 1982) analysis.

For what follows we choose a CCD as the astrometric detector. The noise $\sigma_o$ is the rss of atmospheric, photometric, and DCR noise, viz:

$$\sigma_0 \equiv \left(\frac{\left(\sigma_a^2 + \sigma_p^2\right)}{t} + \sigma_{DCR}^2 + \sigma_{CCD}^2 + \sigma_{ab}^2\right)^{0.5} \qquad \textbf{(11)}$$

where $\sigma_a$ is the atmospheric noise taken from our measurements **($4-5);** $\sigma_p$ is the photometric noise including photon statistics; sky background, and detector noise; $\sigma_{DCR}$ is the DCR; $\sigma_{CCD}$ is any systematic positional error due to CCD spatial non-uniformity; $\sigma_{ab}$ is the positional error due to optical aberrations; and $t$ is the observation time. Thus, atmospheric and photometric noise decrease with time, while the rest are systematic terms.

## 6.2 **DCR Calibration**

In a search for planetary systems, the major systematic error will be DCR. Our experiment at the 5-m attained an accuracy of 0.59 *mas* at tan(z)= 1, compared to the predicted accuracy of 0.430 *mas.* Even if we were to limit observations to near the meridian and add declination constraints, e.g., within 1 hr of meridian crossing and at declinations within 450 degrees of the zenith, the DCR calibration error using the 3-filter calibration method would still be 128 $\mu as$. This is inadequate for a planetary search program that requires random errors of 100 $\mu as$ per source per year (see below). An improvement is to use low resolution spectroscopy. A time-effective method is to place a grating or grism in front of the CCD to perform slitless spectroscopy. Comparing $R_{eff}$ of the spectra of Jacoby et al. (1984) to $R_{eff}$ derived from spectra smoothed to 20 nm, the residuals range from -220 $\mu as$ for a B4 star to 102 $\mu as$ for an MS star, Assuming a worst case of 220 $\mu as$ error in $R_{eff}$, the DCR calibration error is only 66 $\mu as$ when the observations are limited to declinations within450 of zenith and ±1 hr of meridian crossing, We thus adopt $\sigma_{DCR}$ = 66 $\mu as$ as a conservative estimate of the DCR error in the planetary search program.

## 6.3 **Geometric Calibration of the CCD**

Astrometric data analysis on CCDS has been discussed by Monet & Dahn (1983). We determine relative centroid positions of stellar images on the chip to perform relative astrometry. To achieve an astrometric precision of, for example, 100 $\mu as$ requires a centroid accuracy of 0.0024 pixel. This requirement is eased by the fact that the stellar images are spread over many pixels. 0.5 *as* images